Solutions - Midterm Exam

(February 20th @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (20 PTS)

a) Complete the following table. The decimal numbers are unsigned: (5 pts.)

Decimal	BCD	Binary	Reflective Gray Code
36	00110110	100100	110110
50	01010000	110010	101011
128	000100101000	10000000	11000000

b) Complete the following table. The decimal numbers are signed. Use the fewest number of bits in each case: (12 pts.)

	REPRESENTATION		
Decimal	**Sign-and-magnitude**	**1's complement**	**2's complement**
-31	111111	100000	100001
-16	110000	101111	10000
27	011011	011011	011011
-32	1100000	1011111	100000
-1	11	110	1
-19	110011	101100	101101

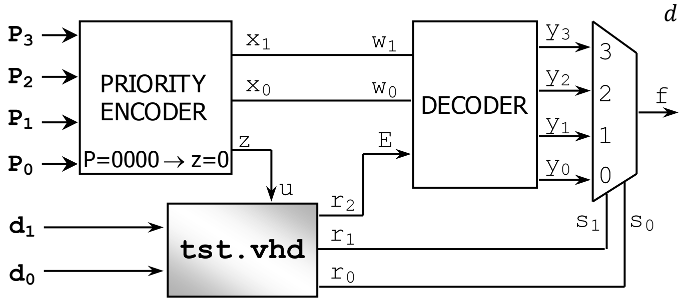
c) Convert the following decimal numbers to their 2's complement representations. (3 pts)

- ✓ -17.125
  +17.125 = 010001.001 ⇒ -17.125 = 101110.111
- ✓ 32.75
  +32.75 = 0100000.11

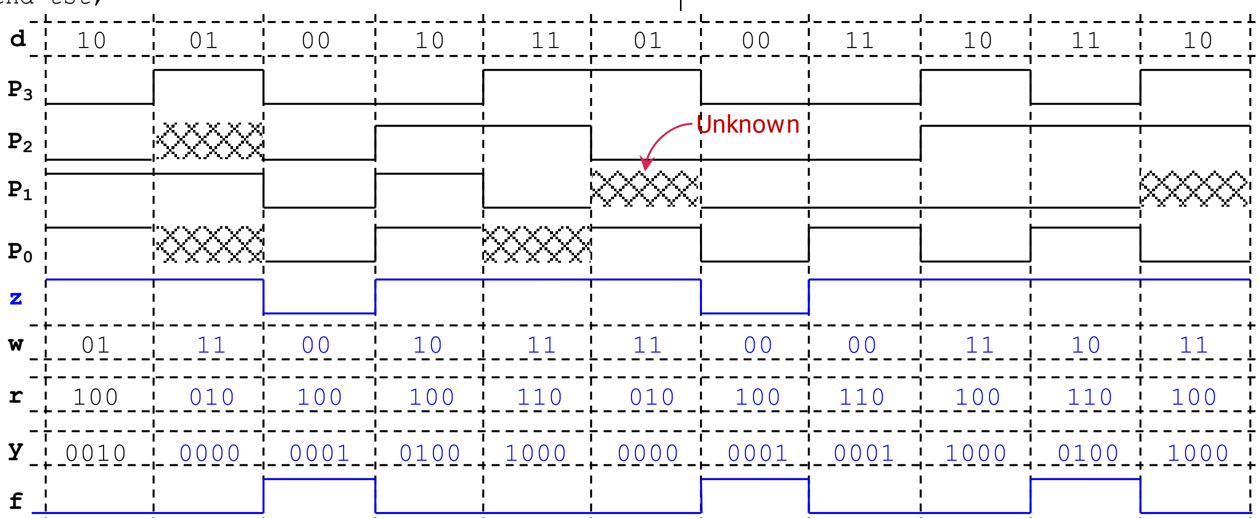
PROBLEM 2 (15 PTS)

- Complete the timing diagram of the following circuit. The VHDL code (`tst.vhd`) corresponds to the shaded circuit.

$d = d_1d_0,\ w = w_1w_0,\ r = r_2r_1r_0,\ y = y_3y_2y_1y_0$

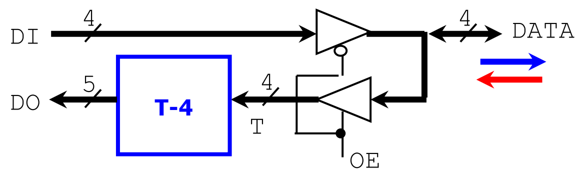


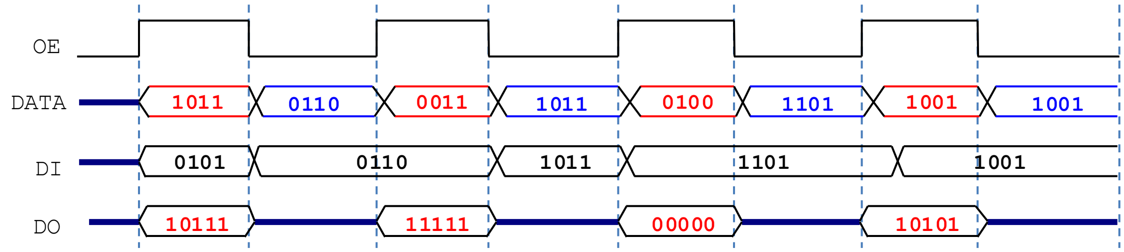

```
library ieee;
use ieee.std_logic_1164.all;
entity tst is
  port (d: in std_logic_vector(1 downto 0);
        r: out std_logic_vector(2 downto 0);
        u: in std_logic);
end tst;

architecture bhv of tst is

begin

  process (d, u)
  begin

     r <= '1'&d;
     if u = '1' then
        r <= d&'0';
     end if;

  end process;

end bhv;
```



PROBLEM 3 (8 PTS)

- Complete the timing diagram (signals *DO* and *DATA*) of the following circuit. The circuit in the blue box computes the signed operation T-4, with the result having 5 bits. T is a 4-bit signed (2C) number.



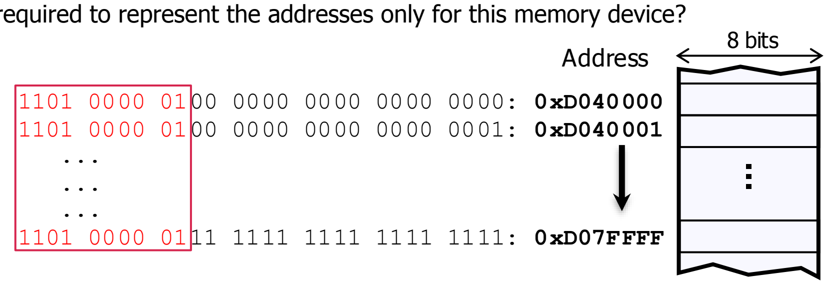


PROBLEM 4 (12 PTS)

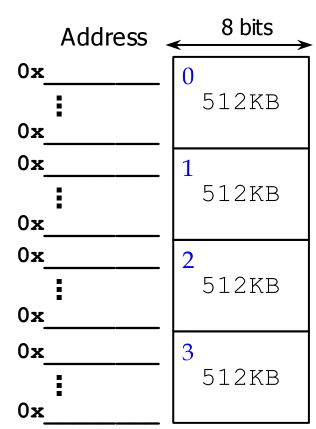
- A microprocessor has a 28-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (5 pts)
  - ✓ What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
    Address Range: `0x0000000` to `0xFFFFFFF`
    With 28 bits, we can address 2^{28} bytes, thus we have $2^8 2^{20}$ = 256 MB of address space

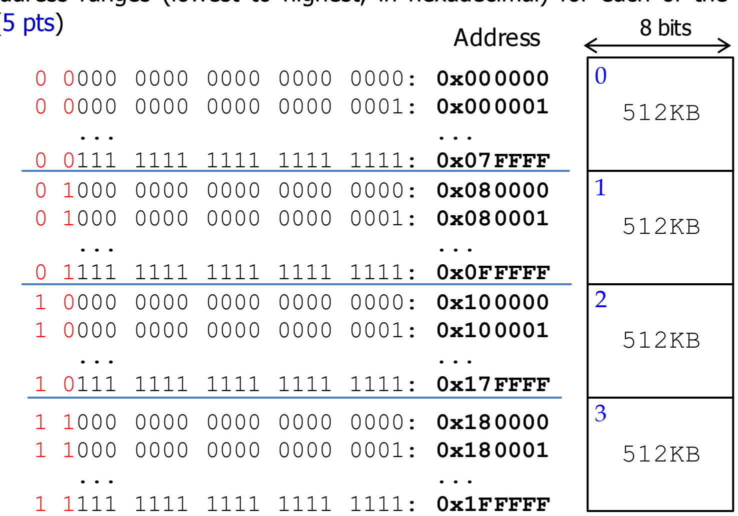
  - ✓ A memory device is connected to the microprocessor. Based on the memory size, the microprocessor has assigned the addresses `0xD040000` to `0xD07FFFF` to this memory device. (3 pts)
    - ▫ What is the size (in bytes, KB, or MB) of this memory device?
    - ▫ What is the minimum number of bits required to represent the addresses only for this memory device?

    As per the figure, we only need 18 bits for the address in the given range (where the memory device is located). Thus, the size of the memory is 2^{18} = 256KB.



- A microprocessor has a memory space of 2 MB. The size of the memory contents of each address is 8 bits (1 byte). (7 pts)
  - ✓ What is the address bus size (number of bits of the address) of this microprocessor?
    Since 2 MB = 2^{21} bytes, the address bus size is 21 bits.
  - ✓ What is the range (lowest to highest, in hexadecimal) of the memory space for this microprocessor?
    With 21 bits, the address range is `0x000000` to `0x1FFFFF`.
  - ✓ The figure (right) shows four memory chips that are placed in the given positions:
    - ▫ Complete the address ranges (lowest to highest, in hexadecimal) for each of the memory chips. (5 pts)





PROBLEM 5 (17 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher byte. (6 pts)

✓ 17 + 50

$c_6=1$ $c_5=1$ $c_4=0$ $c_3=0$ $c_2=0$ $c_1=0$ $c_0=0$

```
50 = 0x32 =   1 1 0 0 1 0 +
17 = 0x11 =   0 1 0 0 0 1
            ---------------
Overflow! → 1 0 0 0 0 1 1
```

✓ 39 - 41

Borrow out! → $b_6=1$ $b_5=1$ $b_4=1$ $b_3=0$ $b_2=0$ $b_1=0$ $b_0=0$

```
39 = 0x27 =   1 0 0 1 1 1 -
41 = 0x29 =   1 0 1 0 0 1
            ---------------
              1 1 1 1 1 0
```

b) Perform the following operations, where numbers are represented in 2's complement. Indicate every carry from c_0 to c_n. For each case, use the fewest number of bits to represent the summands and the result so that overflow is avoided. (8 pts)

✓ -36 + 50

n = 7 bits

$c_7=1$ $c_6=1$ $c_5=1$ $c_4=0$ $c_3=0$ $c_2=0$ $c_1=0$ $c_0=0$

$c_7 \oplus c_6=0$ No Overflow

```
-36 = 1 0 1 1 1 0 0 +
 50 = 0 1 1 0 0 1 0
      ---------------
 14 = 0 0 0 1 1 1 0
```

-36 + 50 = 14 ∈ [-2^6, 2^6-1] → no overflow

✓ -24 - 41

n = 7 bits

$c_7=1$ $c_6=0$ $c_5=0$ $c_4=0$ $c_3=0$ $c_2=0$ $c_1=0$ $c_0=0$

$c_7 \oplus c_6=1$ Overflow!

```
-41 = 1 0 1 0 1 1 1 +
-24 = 1 1 0 1 0 0 0
      ---------------
      0 1 1 1 1 1 1
```

-41 -24 = -65 ∉ [-2^6, 2^6-1] → overflow!

n = 8 bits

$c_8 \oplus c_7=0$ No Overflow

$c_8=1$ $c_7=1$ $c_6=0$ $c_5=0$ $c_4=0$ $c_3=0$ $c_2=0$ $c_1=0$ $c_0=0$

```
-41 = 1 1 0 1 0 1 1 1 +
-24 = 1 1 1 0 1 0 0 0
      -----------------
    1 0 1 1 1 1 1 1 1
```

-41 -24 = -65 ∈ [-2^7, 2^7-1] → no overflow

c) Perform binary multiplication of the following numbers that are represented in 2's complement arithmetic. (3 pts)

✓ -7 x 9

```
  1 0 0 1 x   ➡      1 0 0 1 x
0 1 0 0 1            0 1 1 1
                   ---------
                     1 0 0 1
                   1 0 0 1
                 1 0 0 1
               0 0 0 0
               -------------
               0 1 1 1 1 1 1
                     ⬇
               1 0 0 0 0 0 1
```

PROBLEM 6 (10 PTS)

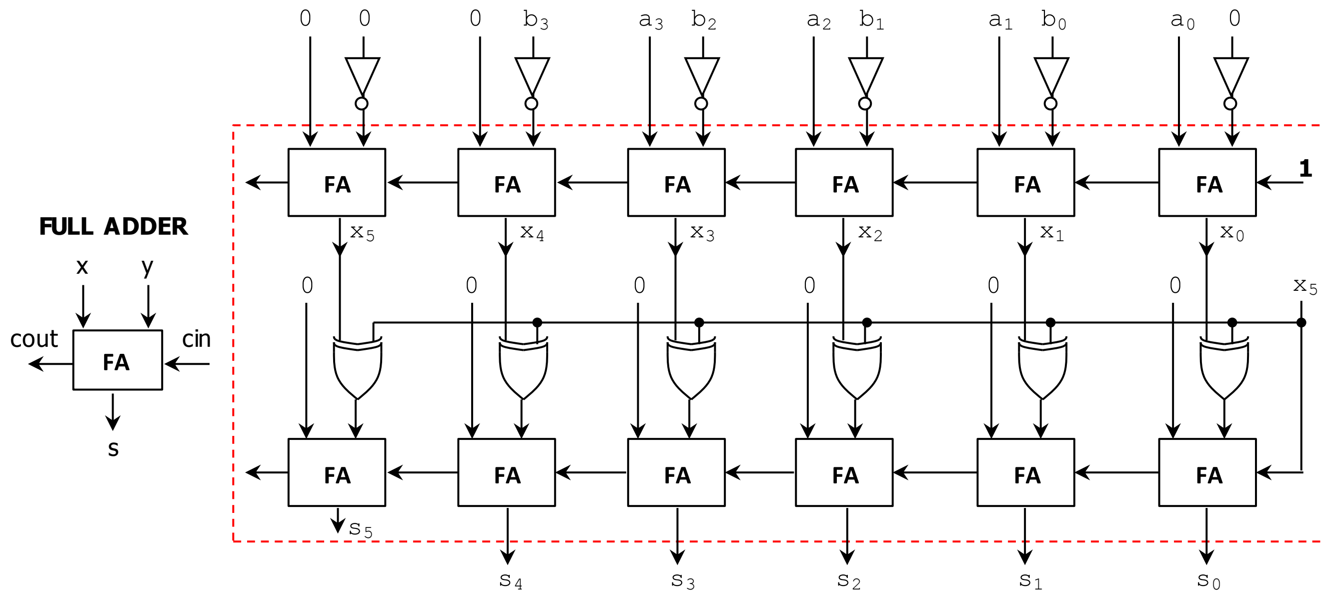
- Given two 4-bit <u>unsigned</u> numbers A, B, sketch the circuit that computes $|A-2B|$. For example: $A = 1010, B = 1110 \rightarrow |A-2B| = |10-2\times 14| = 18$. You can only use full adders and logic gates. Your circuit must avoid overflow: design your circuit so that the result and intermediate operations have the proper number of bits.

$A = a_3a_2a_1a_0$, $B = b_3b_2b_1b_0$: unsigned numbers
$A = 0a_3a_2a_1a_0$, $B = 0b_3b_2b_1b_0$: signed numbers (2C)

$A, B \in [0,15] \rightarrow 2B \in [0,30]$ requires 6 bits in 2C.

✓ $X = A - 2B \in [-30,15]$ requires 6 bits in 2C. Thus, the operation $A-2B$ requires 6 bits (we sign-extend A).
✓ $|X| = |A-2B| \in [0,30]$ requires 6 bits in 2C. Thus, the second operation $0 \pm X$ only requires 6 bits.
  - If $x_5 = 1 \rightarrow X < 0 \rightarrow$ we do $0 - X$.
  - If $x_5 = 0 \rightarrow X \geq 0 \rightarrow$ we do $0 + X$.

✓ $|X| = |A-2B| \in [0,30]$ requires 6 bits in 2C. Note that the MSB is always 0. The unsigned result only requires 6 bits.



PROBLEM 7 (18 PTS)

- Sketch the circuit that implements the following Boolean function: $f(a,b,c,d) = (c\oplus d)\left(\overline{a\oplus b}\right)$
  - ✓ Using ONLY 2-to-1 MUXs (AND, OR, NOT, XOR gates are not allowed). (12 pts)

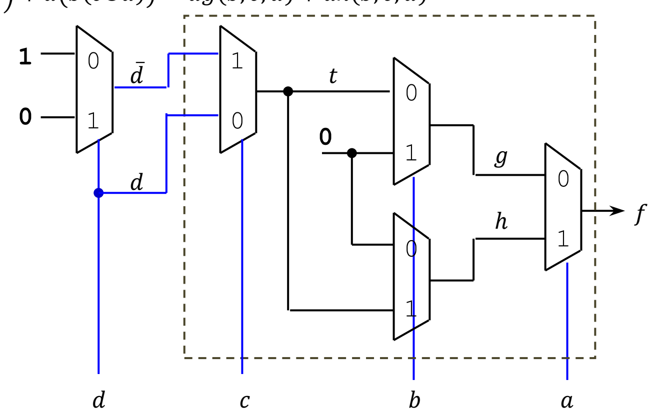
$f(a,b,c,d) = \bar{a}f(0,b,c,d) + af(1,b,c,d) = \bar{a}\left(\bar{b}(c\oplus d)\right) + a\left(b(c\oplus d)\right) = \bar{a}g(b,c,d) + ah(b,c,d)$

$g(b,c,d) = \bar{b}g(0,c,d) + bg(1,c,d) = \bar{b}(c\oplus d) + b(0)$

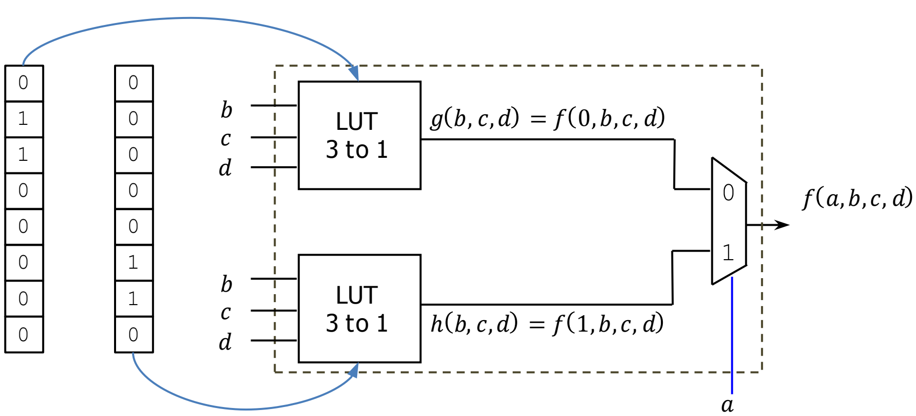
$h(b,c,d) = \bar{b}h(0,c,d) + bh(1,c,d) = \bar{b}(0) + b(c\oplus d)$

$t(c,d) = c\oplus d = \bar{c}t(0,d) + ct(1,d) = \bar{c}(d) + c(\bar{d})$

Also: $\bar{d} = \bar{d}(1) + d(0)$



  - ✓ Using two 3-to-1 LUTs and a 2-to-1 MUX. Specify the contents of each of the 3-to-1 LUTs. (6 pts)



a	b	c	d	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0